

Removal of angular momentum by strong magnetic field stresses in advective accretion flows around black holes

Banibrata Mukhopadhyay

Department of Physics, Indian Institute of Science, Bangalore 560012
E-mail: bm@physics.iisc.ernet.in

Koushik Chatterjee

Department of Physics, Indian Institute of Technology Kharagpur, Kharagpur 721302
E-mail: kchatterjee009@gmail.com

We show that the removal of angular momentum is possible in the presence of large scale magnetic stresses, arisen by fields much stronger than that required for magnetorotational instability, in geometrically thick, advective, sub-Keplerian accretion flows around black holes in steady-state, in the complete absence of alpha-viscosity. The efficiency of such angular momentum transfer via Maxwell stress, with the field well below its equipartition value, could be equivalent to that of alpha-viscosity, arisen via Reynolds stress, with $\alpha = 0.01 - 0.08$. We find in our simpler vertically averaged advective disk model that stronger the magnetic field and/or larger the vertical-gradient of azimuthal component of magnetic field, stronger the rate of angular momentum transfer is, which in turn may lead to a faster rate of outflowing matter, which has important implications to describe the hard spectral states of black hole sources. When the generic origin of alpha-viscosity is still being explored, mechanism of efficient angular momentum transfer via magnetic stresses alone is very interesting.

Keywords: accretion, accretion disks; MHD; jets and outflows; X-rays: binaries; galaxies: active

1. Introduction

Blandford and Payne¹ showed angular momentum transfer via large magnetic stresses, in the absence of alpha-viscosity, in the framework of self-similar Keplerian disk flows. Here, we show in a simpler 1.5-dimensional, vertically averaged disk model that the Maxwell stresses due to strong magnetic fields are adequate enough for angular momentum transfer even in advective accretion flows, without self-similar assumption, describing the hard spectral states of black hole sources.

The idea of exploring magnetic stress in order to explain astrophysical systems is not really new. This was implemented, for example, in the solar wind which was understood to have decreased Sun's angular momentum through the effect of magnetic stresses (see, e.g., Ref. 2), in the proto-stellar gas clouds which might have been contracted by magnetic effects³. Ozernoy and Usov⁴ and Blandford⁵ showed that the energy is possible to extract continuously by electromagnetic torques and twisted field lines in accretion disks. By linear stability analysis of the accretion disks, it was shown by Cao and Spruit⁶ that angular momentum is possible to remove by the magnetic torque exerted by a centrifugally driven wind. However, by solving the local vertical structure of a geometrically thin accretion disk threaded by a poloidal magnetic field, Ogilvie and Livio⁷ showed the shortcoming of launching

an outflow and suggested for an existence of additional source of energy for its successful launching.

Here we demonstrate, semi-analytically, the effects of strong magnetic field, stronger than that needed for magnetorotational instability (MRI)⁸, with plasma- $\beta > 1$ yet, on to the vertically averaged advective accretion flows in vertical equilibrium in order to transport matter. Therefore, we consider the flow variables to depend on the radial coordinate only. Although, in reality, a non-zero vertical magnetic field should induce a vertical motion, in the platform of the present assumption, any vertical motion will be featured as an outward motion. Indeed, our aim here is to furnish removal of angular momentum from the flow via magnetic stresses, independent of its vertical or outward transport.

2. Basic model equations

We describe optically thin, magnetized, viscous, axisymmetric, advective, vertically averaged, steady-state accretion flow, in the pseudo-Newtonian framework with the Mukhopadhyay⁹ potential. Hence, the equation of continuity, vertically averaged hydromagnetic equations for energy-momentum balance in different directions are given by (assuming that the dimensionless variables do not vary significantly in the vertical direction such that $\partial/\partial z \sim s_i/h$ and, as a consequence, the vertical component of velocity is zero),

$$\begin{aligned} \dot{M} &= 4\pi x \rho h \vartheta, \quad \vartheta \frac{d\vartheta}{dx} + \frac{1}{\rho} \frac{dP}{dx} - \frac{\lambda^2}{x^3} + F = \frac{1}{4\pi\rho} \left(B_x \frac{dB_x}{dx} + s_1 \frac{B_x B_z}{h} - \frac{B_\phi^2}{x} \right), \\ \vartheta \frac{d\lambda}{dx} &= \frac{1}{x\rho} \frac{d}{dx} (x^2 W_{x\phi}) + \frac{x}{4\pi\rho} \left(B_x \frac{dB_\phi}{dx} + s_2 \frac{B_z B_\phi}{h} + \frac{B_x B_\phi}{x} \right), \\ \frac{P}{\rho h} &= \frac{Fh}{x} - \frac{1}{4\pi\rho} \left(B_x \frac{dB_z}{dx} + s_3 \frac{B_z^2}{h} \right), \\ \vartheta T \frac{ds}{dx} &= \frac{\vartheta}{\Gamma_3 - 1} \left(\frac{dP}{dx} - \frac{\Gamma_1 P}{\rho} \frac{d\rho}{dx} \right) = Q^+ - Q^- = Q_{vis}^+ + Q_{mag}^+ - Q_{vis}^- - Q_{mag}^-, \end{aligned} \quad (1)$$

where $W_{x\phi} = \alpha(P + \rho\vartheta^2)$ and α the Shakura-Sunyaev viscosity parameter¹⁰. Note that s_1 , s_2 and s_3 are the degrees of vertical scaling for the radial, azimuthal and vertical components of the magnetic field respectively. Here \dot{M} is the conserved mass accretion rate, ρ the mass density of the flow, ϑ the radial velocity, P the total pressure including the magnetic contribution, F the force corresponding to the pseudo-Newtonian potential for rotating black holes⁹, λ the angular momentum per unit mass, $W_{x\phi}$ the viscous shearing stress written following the Shakura-Sunyaev prescription¹⁰ with appropriate modification¹¹, $h \sim z$, the half-thickness and x the radial coordinate of the disk, when both of them are expressed in units of GM/c^2 , where G the Newton's gravitation constant, M the mass of black hole, c the speed of light, s the entropy per unit volume, T the (ion) temperature of the flow, Q^+ and Q^- are the net rates of energy released and radiated out per unit volume in/from the

flow respectively (when Q_{vis}^+ , Q_{mag}^+ , Q_{vis}^- , Q_{mag}^- are the respective contributions from viscous and magnetic parts). All the variables are made dimensionless in the spirit of dimensionless x and z . We further assume, for the present purpose, the heat radiated out proportional to the released rate with the proportionality constants $(1 - f_{vis})$ and $(1 - f_m)$, respectively, for viscous and magnetic parts of the radiations. Γ_1 , Γ_3 , which are functions of polytropic constant γ , indicate the polytropic indices depending on the gas and radiation content in the flow (see, e.g., Ref. 12, for exact expressions) and B_x , B_ϕ and B_z are the components of magnetic field. The model for Q_{vis}^+ is taken from the previous work¹², and the relation for Q_{mag}^+ is taken from that by Bisnovatyi-Kogan and Ruzmaikin¹³.

Hydromagnetic flow equations must be supplemented by (for the present purpose, steady-state) equations of induction and no magnetic monopole at the limit of very large Reynolds number, given by

$$\nabla \times \vec{v} \times \vec{B} = 0, \quad \frac{d}{dx}(xB_x) + s_3 \frac{B_z}{h} = 0, \quad (2)$$

when \vec{v} and \vec{B} are respectively the velocity and magnetic field vectors and ν_m is the magnetic diffusivity.

3. Solutions and Results

We take into account two situations. (1) Flows with a relatively higher \dot{M} and, hence, lower γ , modelled around stellar mass black holes: such flows may or may not form Keplerian accretion disks. (2) Flows with a lower \dot{M} and, hence, higher γ , modelled around supermassive black holes: such flows are necessarily hot gas dominated advective (or advection dominated) accretion flows.

We find that the flows with plasma- $\beta > 1$, but $\alpha = 0$, exhibit adequate matter transport, as efficient as the α -viscosity with $\alpha = 0.08$, but without magnetic stresses, would do. This is interesting as the origin of α (and the corresponding instability and turbulence) is itself not well understood. The maximum required large scale magnetic field is $\sim 10^5$ G in a disk around $10M_\odot$ black holes and ~ 10 G in a disk around $10^7 M_\odot$ supermassive black holes, where M_\odot is solar mass. The presence of such a field, in particular for a stellar mass black hole disk when the binary companion supplying mass is a Sun-like star with the magnetic field on average 1G, may be understood, if the field is approximately frozen with the disk fluids (or the supplied fluids from the companion star remain approximately frozen with the magnetic field) or disk fluids exhibit very large Reynolds number. Indeed, all the present computations are done at the limit of large Reynolds number, as really is the case in accretion flows, such that the term associated with the magnetic diffusivity in the induction equation is neglected. The size of a disk around supermassive black holes is proportionately larger compared to that around a stellar mass black hole. Hence, from the equipartition theory, indeed the magnetic field is expected to be decreased here compared to that around stellar mass black holes. Figure 1 shows

a typical set of accretion solutions and confirms that flows around a stellar mass black hole with α -viscosity but without large scale magnetic fields (viscous flow) show similar behavior to that with large scale magnetic fields without α -viscosity (magnetic flow). The flows around a supermassive black hole show very similar features, except with reduced field strength. For other details, see Ref. 14.

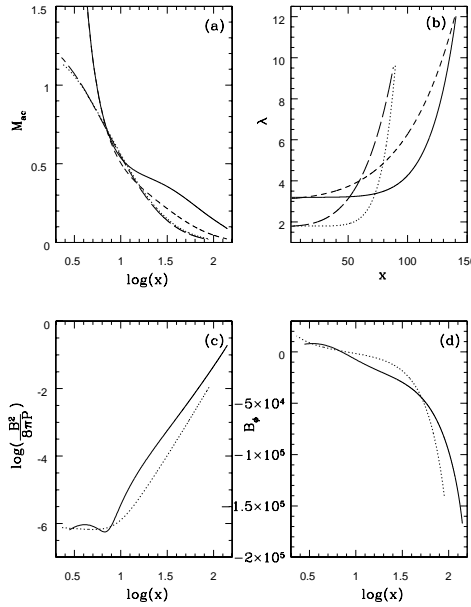


Fig. 1. (a) Mach number, (b) angular momentum per unit mass in GM/c , (c) inverse of plasma- β , (d) azimuthal component of magnetic field in G, when solid and dotted lines are for magnetic flows around Schwarzschild ($a = 0, \lambda_c = 3.2$) and Kerr ($a = 0.998, \lambda_c = 1.8$) black holes respectively, and dashed and long-dashed lines are for viscous flows around Schwarzschild ($a = 0, \alpha = 0.017, \lambda_c = 3.15$) and Kerr ($a = 0.998, \alpha = 0.012, \lambda_c = 1.8$) black holes respectively, where a is the dimensionless spin parameter of black hole and λ_c the quantity at critical radius. Other parameters are $M = 10M_\odot$, $\dot{M} = 0.1$ Eddington rate, polytropic constant $\gamma = 1.335$, $f_{vis} = f_m = 0.5$, $s_2 = -0.5$.

Let us now explore in more details, how exactly various components of magnetic stress lead to angular momentum transfer in the flows. Figure 2a shows that the stress component $B_x B_z$ around a Schwarzschild black hole increases almost throughout as matter advances towards the black hole. This implies that the flow is prone to outflow through the field lines, which effectively helps in infalling matter towards the black hole. However, in the near vicinity of black hole, $B_x B_z$ decreases, as indeed outflow is not possible therein. In this flow zone, the angular momentum becomes very small which practically does not affect the infall. The magnitude of $B_\phi B_z$ decreases till the inner region of accretion flow, implying the part of matter

to be spiralling out and, hence, removing angular momentum leading to infall of matter. Finally, the magnitude of $B_x B_\phi$ increases at a large and a small distances from the black hole (except around the transition radius), which helps removing angular momentum and further infall. This is the same as the Shakura-Sunyaev viscous stress would do with the increase of matter pressure. However, at the intermediate zone, the angular momentum transfer through $B_x B_\phi$ reverses and a part of the matter outflows. At the Keplerian to sub-Keplerian transition zone, due to the increase of flow thickness, matter is vertically kicked effectively, showing a decrease of $B_x B_\phi$. Majority of the features remain similar for the flow around a rotating black hole, as shown in Fig. 2b. However, a rotating black hole reveals a stronger/efficient outflow/jet in general. Hence, except at the inner zone, $B_x B_\phi$ decreases throughout, which helps kicking the matter outwards by transferring the angular momentum inwards.

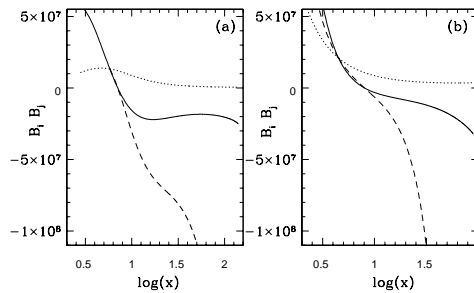


Fig. 2. Components of magnetic stress: $B_x B_\phi$ (solid line), $B_x B_z$ (dotted line), $B_\phi B_z$ (dashed line), for (a) Schwarzschild magnetic flow of Fig. 1, (b) Kerr magnetic flow of Fig. 1.

Different components of the magnetic stress tensor have different roles: $B_x B_\phi$ controls the infall in the disk plane, whereas $B_\phi B_z$ renders the flow to spiral outwards and, hence, outflow. Moreover, $B_x B_z$ helps to kick the matter out vertically. Larger the field strength, larger is the power of magnetic stresses. Interestingly, the magnitude of magnetic field decreases, as the steady-state matter advances towards the black hole. This is primarily because $B_\phi B_z$ (and also $B_x B_\phi$ for a rotating black hole) decreases inwards almost entirely in order to induce inflow via angular momentum transfer through outflow. This further reveals a decreasing $|B_\phi|$ as the output of self-consistent solutions of the coupled set of equations.

4. Discussion and Conclusions

Is there any observational support for the existence of such a magnetic field, as required for the magnetic accretion flows discussed here? Interestingly, the polarization measurements in the hard state of Cyg X-1 imply that it should have at least

10mG field at the source of emission¹⁵. In order to explain such high polarization, a jet model was suggested by Zdziarski et al.¹⁶, which requires a magnetic field $\sim (5 - 10) \times 10^5 \text{G}$ at the base of jet and hence in the underlying accretion disk.

In the present computations, we have assumed the flow to be vertically averaged without allowing any vertical component of the flow velocity. The most self-consistent approach, in order to understand vertical transport of matter through the magnetic effects which in turn leads to the radial infall of rest of the matter, is considering the flow to be moving in the vertical direction from the disk plane as well. Such an attempt, in the absence of magnetic and viscous effects, was made earlier by one of the present authors¹⁷ in the model framework of coupled disk-outflow systems. In such a framework, the authors further showed that the outflow power of the correlated disk-outflow systems increases with the increasing spin of black holes. Our future goal is now to combine that model with the model of present work, so that the coupled disk-outflow systems can be investigated more self-consistently and rigorously, when the magnetic field plays indispensable role in order to generate vertical flux in the three-dimensional flows.

Acknowledgments

K.C. thanks the Academies' Summer Research Fellowship Programme of India for offering him a Fellowship to pursue his internship in Indian Institute of Science, Bangalore, when most of the calculations of this project were done.

References

1. R.D. Blandford and D.G. Payne, *MNRAS* **199**, 883 (1982).
2. E.J. Weber and L. Davis Jr., *Astrophys. J* **148**, 217 (1967).
3. T.C. Mouschovias and E.V. Paleologou, *M&P* **22**, 31 (1980).
4. L.M. Ozernoy and V.V. Usov, *Astrophys. Sp. Sc.* **25**, 149 (1973).
5. R.D. Blandford, *MNRAS* **176**, 465 (1976).
6. X. Cao and H.C. Spruit, *Astron. Astrophys.* **385**, 289 (2002).
7. G. Ogilvie and M. Livio, *Astrophys. J* **499**, 329 (1998).
8. S.K. Nath and B. Mukhopadhyay, *Phys. Rev. E* **92**, 023005 (2015).
9. B. Mukhopadhyay, *Astrophys. J* **581**, 427 (2002).
10. N. Shakura and R. Sunyaev, *Astron. Astrophys.* **24**, 337 (1973).
11. B. Mukhopadhyay and S. Ghosh, *MNRAS* **342**, 274 (2003).
12. S.R. Rajesh and B. Mukhopadhyay, *MNRAS* **402**, 961 (2010).
13. G.S. Bisnovatyi-Kogan and A.A. Ruzmaikin, *Astrophys. Sp. Sc.* **28**, 45 (1974).
14. B. Mukhopadhyay and K. Chatterjee, *Astrophys. J* **807**, 43 (2015).
15. P. Laurent, et al., *Science* **332**, 438 (2011).
16. A.A. Zdziarski, P. Pjanka, M. Sikora and L. Stawarz, *MNRAS* **442**, 3243 (2014).
17. D. Bhattacharya, S. Ghosh and B. Mukhopadhyay, *Astrophys. J* **713**, 105 (2010).